L12: Invariants and Specification Inference

Software Analysis

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Invariants are what make pre- and post-conditions of code containing loops and objects provable.

Naik, *Software Specifications*, CIS 5470, Penn

Black Board

Quiz: Loop Invariants

Which of the following are *valid* loop invariants, and which are strong enough to prove the post-condition?

```
i = 0; s = 0;
while (i != n) { s = s + a[i]; i = i + 1; }
// post:  s == a[0] + ...  + a[n-1]
```

1. `i == i`
2. `0 <= i && i <= n`
3. `s == a[0] + ...  + a[i-1]`
4. `s >= 0`

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3. `s == a[0] + ... + a[i-1]` — **valid** and **sufficient**: with $i = n$ at exit it gives the post-condition
4. `s >= 0` — **not valid**: it fails if the array contains negative elements

Validity is not enough: a loop has infinitely many valid invariants, and the useful one is the one that implies the post-condition.

Example: Class Invariant

```
class Rational {
    //@invariant denom != 0;
    int num, denom;

    //@requires d != 0;
    Rational(int n, int d) { num = n; denom = d; }

    double getDouble() { return ((double) num) / denom; }

    public static void main(String[] a) {
        int n = readInt(), d = readInt();
        if (d == 0) return;
        Rational r = new Rational(n, d);
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```

The class invariant makes the absence of a division by zero in `getDouble()` provable.

Quiz: Class Invariants

Is `count >= 0` a class invariant of this class? If not, what is the smallest change that makes it one?

```
class Counter {
    //@invariant count >= 0;
    int count;

    Counter()       { count = 0; }
    void inc()      { count = count + 1; }
    void dec()      { count = count - 1; }
    int  get()      { return count; }
}
```

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Solution: Class Invariants

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- `inc()` and `get()` **preserve** it
- `dec()` **breaks** it: calling it on a counter with `count == 0` yields -1

Two ways to repair the class:

```
//@requires count > 0;            // oblige the caller
void dec() { count = count - 1; }

void dec() { if (count > 0) count = count - 1; }    // guard in the method
```

Inferring Specifications

Motivation and Approach

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- Programmers are reluctant to write and maintain specifications
- An idea: infer them automatically
- Many approaches to **specification mining**, with varying trade-offs
- A popular guess-and-check approach: the **Houdini algorithm**

  requires an automated theorem prover

Houdini Algorithm

- An annotation assistant for the static modular verifier **ESC/Java**

Flanagan & Leino, *Houdini, an Annotation Assistant for ESC/Java*, FME 2001 · Naik, *Software Specifications*, CIS 5470, Penn

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- An annotation assistant for the static modular verifier **ESC/Java**
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Houdini Algorithm

- An annotation assistant for the static modular verifier **ESC/Java**
- Generate many **candidate invariants**; use ESC/Java to verify or refute each
- Candidates may be guessed by

  static analysis — mined from the source code using heuristics

  dynamic analysis — facts observed while running the program, as in **Daikon**

Flanagan & Leino, *Houdini, an Annotation Assistant for ESC/Java*, FME 2001 · Naik, *Software Specifications*, CIS 5470, Penn

Houdini: Example Candidate Invariants

For `int i, j`, with `cmp` $\in \{<, <=, ==, !=, >, >=\}$:

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//@ invariant i cmp j;
//@ invariant i cmp 0;
```

Houdini: Example Candidate Invariants

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For `Object[] a`:

```
//@ invariant a != null;
//@ invariant a.length cmp i;
//@ invariant (forall int k; 0 <= k && k < a.length ==> a[k] != null);
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For `Object[] a`:

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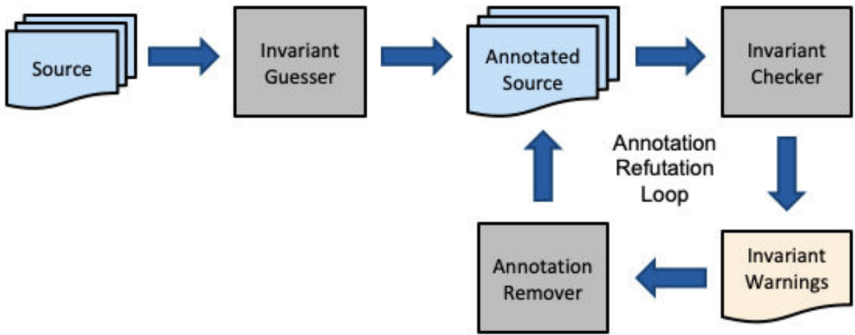
//@ invariant a != null;
//@ invariant a.length cmp i;
//@ invariant (forall int k; 0 <= k && k < a.length ==> a[k] != null);
```

For `boolean f`:

```
//@ invariant f == false;
//@ invariant f == true;
```

Workflow of Houdini



Houdini Algorithm

Input: An unannotated program P

Output: An annotated version of P

Algorithm:

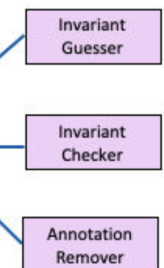
Generate set of candidate invariants and put into P;

repeat

invoke the invariant checker to check P;

remove any refuted candidate invariants from P;

until no warnings remain;



Example: Houdini on a Buggy Program

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The four annotations in blue are **guesses**; the guard checks the wrong variable.

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Example: Houdini on the Corrected Program

```
class Rational {
    //@invariant num != 0; // removed
    //@invariant denom != 0; // kept
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No warnings remain, and the surviving class invariant proves the division safe.

Quiz: Inferring Contracts Using Houdini

Which pre- and post-conditions does Houdini infer for `foo` and `bar`?

```
main() {
    foo(5, 0);
}
```

```
foo(x, y) {
    if (x <= 0) z := y;
    else z := bar(x, y);
    return z;
}
```

```
bar(x, y) {
    x := x - 1;
    y := y + 1;
    return foo(x, y);
}
```

	candidate pre-conditions				candidate post-conditions	
	`x >= 0`	`y >= 0`	`x == y`	`x > 0`	`ret >= 0`	`ret == 0`
`foo`						
`bar`						

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`foo`	yes	yes	–	–		
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- `foo`: `x == y` fails at `foo(5, 0)`; `x > 0` fails at the call in `bar`, after `x := x - 1`

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- `foo`: `x == y` fails at `foo(5, 0)`; `x > 0` fails at the call in `bar`, after `x := x - 1`
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`foo`	yes	yes	–	–	yes	–
`bar`	–	yes	–	yes	yes	–

- `foo`: `x == y` fails at `foo(5, 0)`; `x > 0` fails at the call in `bar`, after `x := x - 1`
- `foo`: `ret >= 0` holds on both branches; `ret == 0` fails
- `bar` is called only where `x > 0`, and returns `foo(x, y)`

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- High-quality invariants require many candidates, which makes refutation expensive
- No guarantee that the inferred invariants verify the property of interest

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Finding Inductive Loop Invariants using Large Language Models

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ADITYA SENTHILNATHAN*, Cornell, USA
SAIKAT CHAKRABORTY, Microsoft Research, USA
PANTAZIS DELIGIANNIS, Microsoft Research, USA
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