L17: Data-Flow Analysis

Software Analysis

IIT Guwahati · August 8, 2026

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- Different kinds of data: **constants**, **variables**, **expressions**.
- Used by **bug-finding tools** and by **compilers**.
- *How:* propagate facts along the edges of a control-flow graph; for loops, **iterate to a fixed point**.

The WHILE Language

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x = 5;
y = 1;
while (x != 1) {
    y = x * y;
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Computes the factorial of 5 in `x`.

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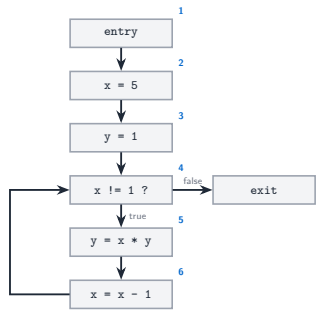
- No functions, pointers or threads.
- Loops alone already make interesting properties **undecidable**.

The Control-Flow Graph

A directed graph that summarising the flow of control in *all* possible runs of the program.



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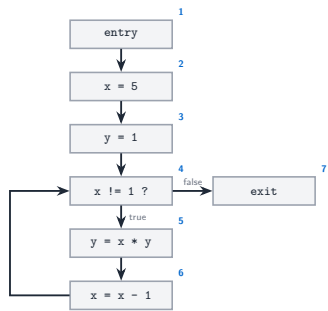

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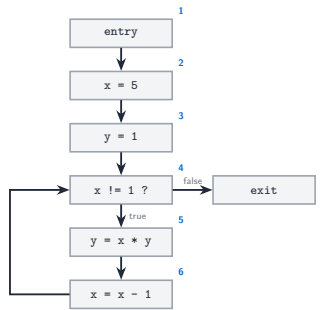

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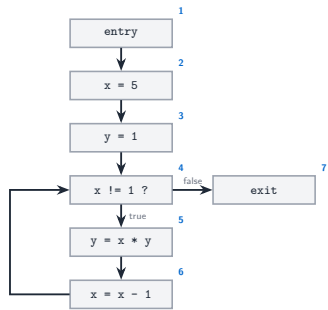

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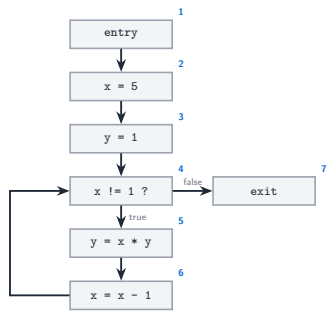

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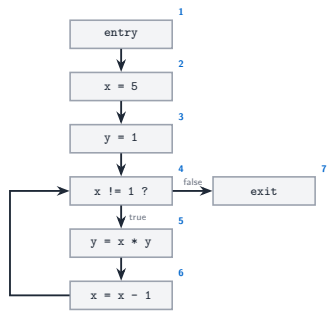

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- **Sound** $\Rightarrow$ it reports *all* facts that can occur in a real run.
- **Incomplete** $\Rightarrow$ it may report facts that can *never* occur.

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- Conditions are abstracted by **non-deterministic choice** $*$.
- Each branch is assumed to go *either* way.
- **All real paths are covered** — **and some impossible ones too**.

Classic Data-Flow Analyses

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Reaching Definitions

find uninitialised variable uses

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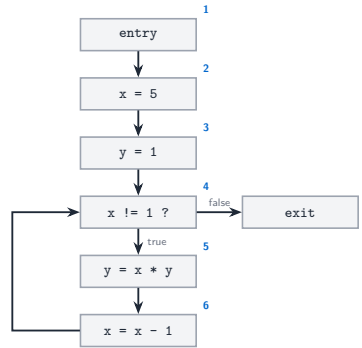
Goal: Which Assignments Might Still Hold?

A definition **reaches** a program point p if, along *some* path to p, the assignment has been made and not overwritten.



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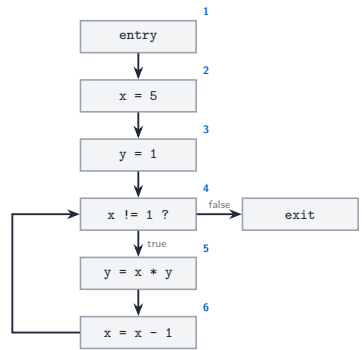

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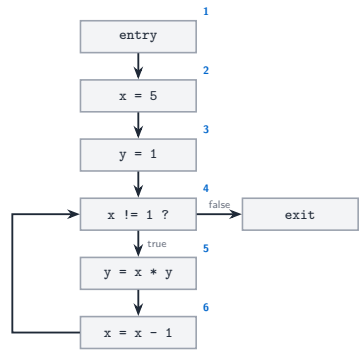

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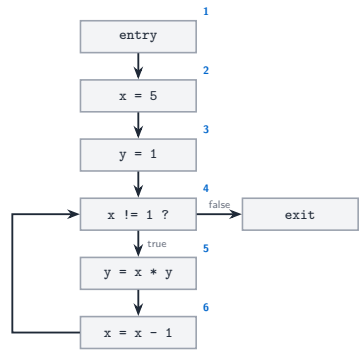

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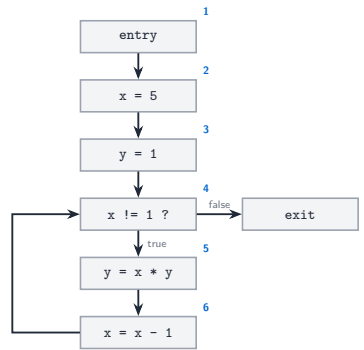
- `x = 5` **does not reach P2**: node 6 overwrites `x` every time.



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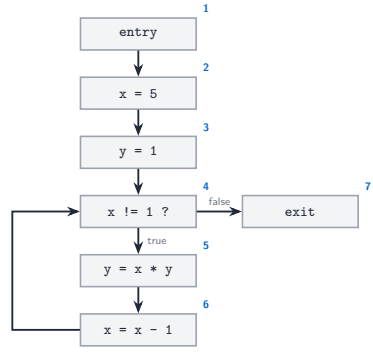

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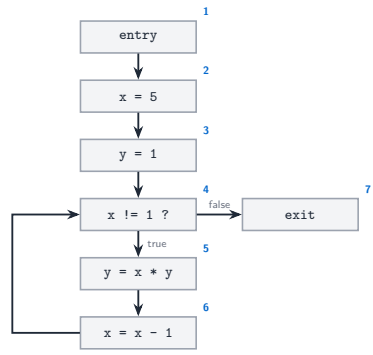

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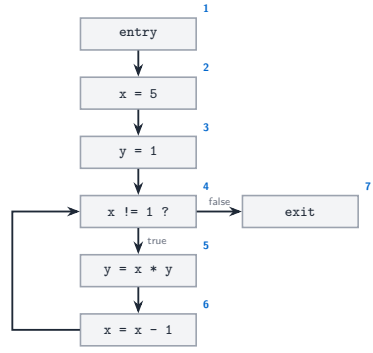

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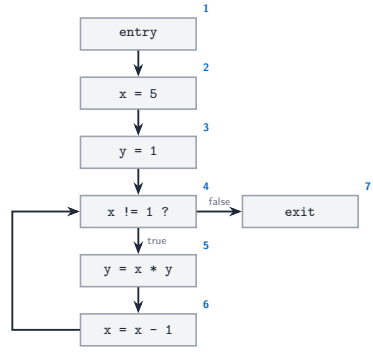

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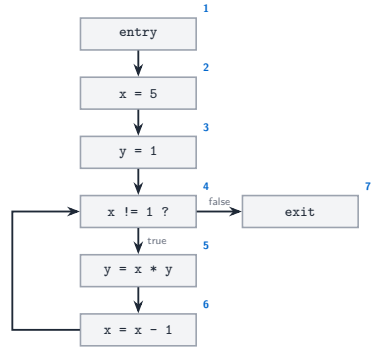

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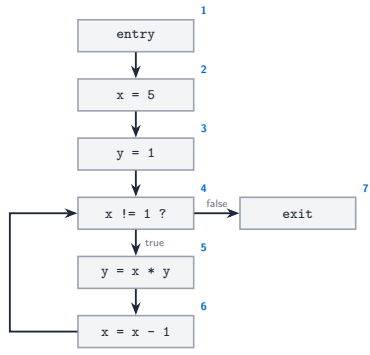

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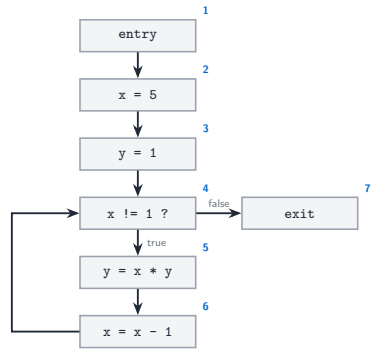

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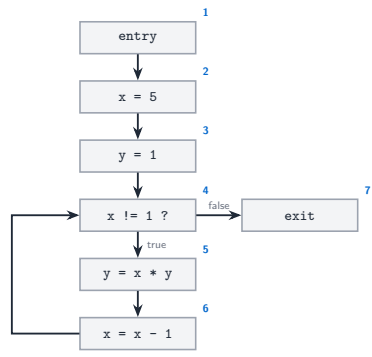

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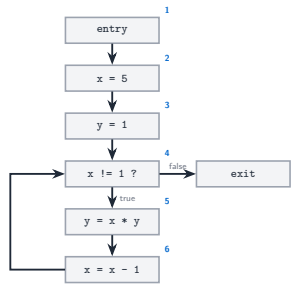

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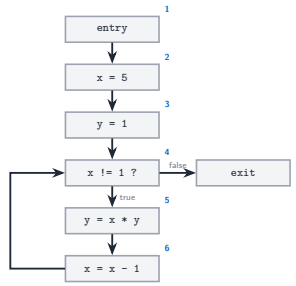

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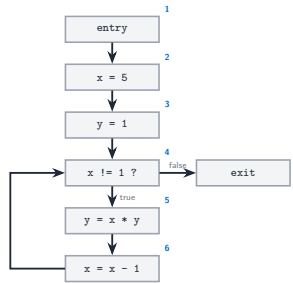

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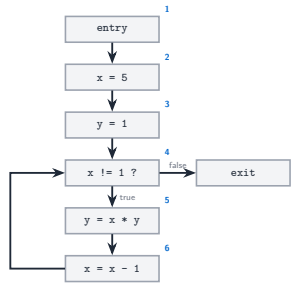

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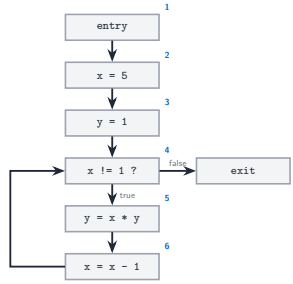
- That point is called **saturated**, or a **fixed point**.



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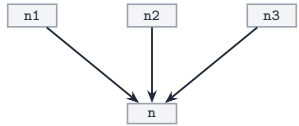

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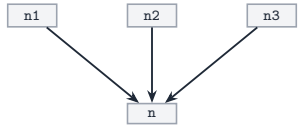

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- Unlike Operation #1, this **depends on the statement** at n.

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An assignment to x kills **every other** definition of x — including $\langle \mathtt{x}, ? \rangle$.

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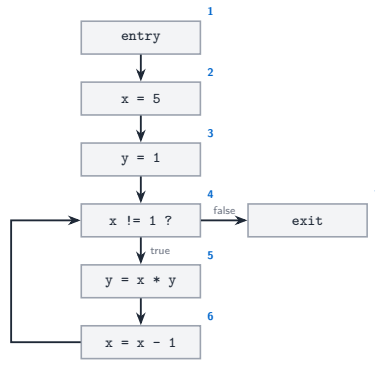
A condition changes nothing: $\mathrm{OUT}[n] = \mathrm{IN}[n]$.

An assignment to x kills **every other** definition of x — including $\langle \mathtt{x}, ? \rangle$.

GEN and KILL for Our Program

n	Statement	GEN	KILL
2	`x = 5`		
3	`y = 1`		
4	`x != 1`		
5	`y = x * y`		
6	`x = x - 1`		



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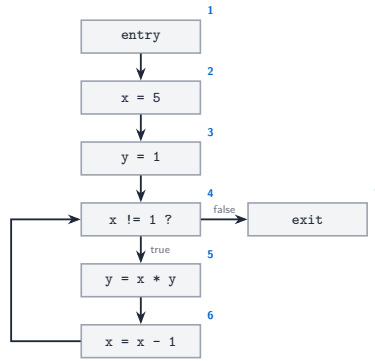

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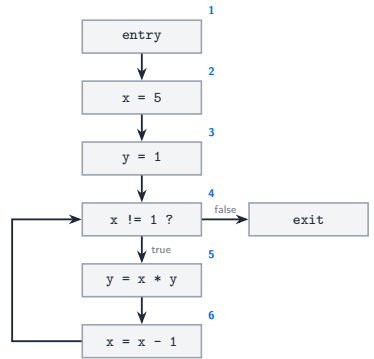

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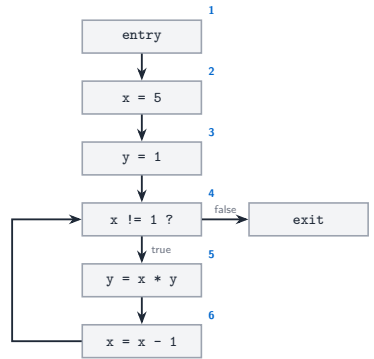

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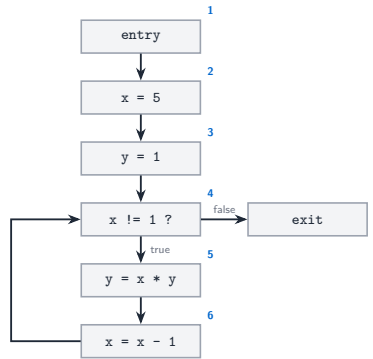

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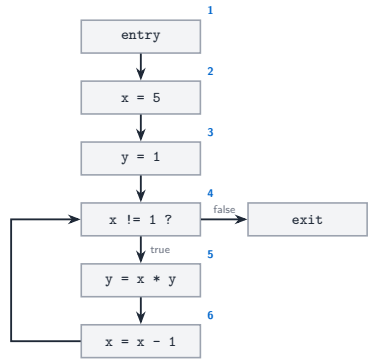

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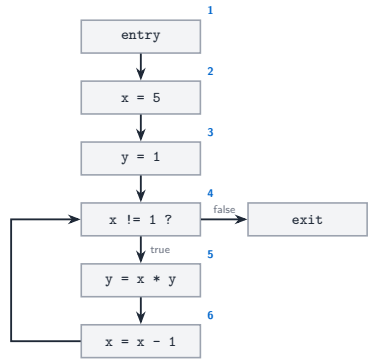
$\langle x, ?\rangle$ and $\langle y, ?\rangle$ are **hypothetical** definitions at the program entry: "uninitialised at the start".



Chaotic Iteration Algorithm

1. **Initialise:** Set $\mathrm{IN}[n]$ and $\mathrm{OUT}[n]$ to $\emptyset$ for every node n.
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$$\mathrm{OUT}[1] = \{\langle x, ?\rangle, \langle y, ?\rangle\}.$$
3. **Repeat the following until convergence:**
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Our program. $S = 2^D$, the sets of definitions, with $A \sqsubseteq B$ meaning $A \subseteq B$.

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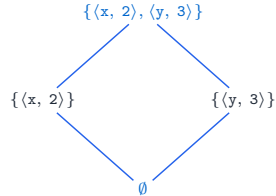

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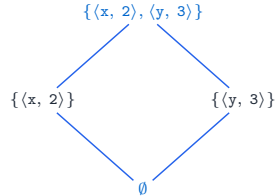

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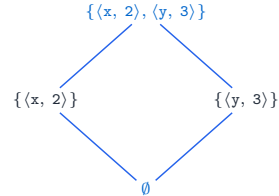

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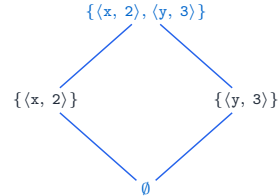

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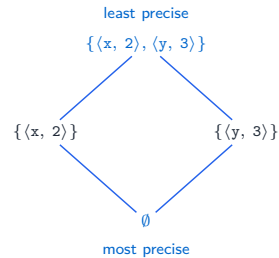

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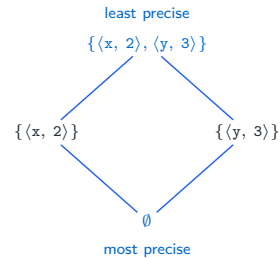

$$\{\langle \mathrm{x},\ 2\rangle\} \sqcup \{\langle \mathrm{y},\ 3\rangle\} = \{\langle \mathrm{x},\ 2\rangle, \langle \mathrm{y},\ 3\rangle\} \quad \text{(LUB / Join)}$$

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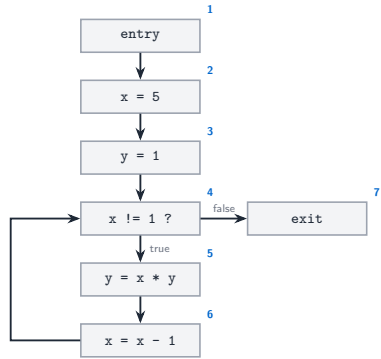
The Abstract Domain for Reaching Definitions

- Definitions: $\langle x, 2\rangle$, $\langle y, 3\rangle$, $\langle y, 5\rangle$, $\langle x, 6\rangle$.



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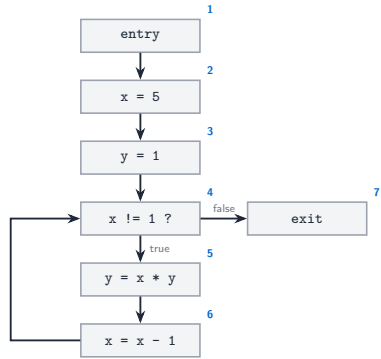

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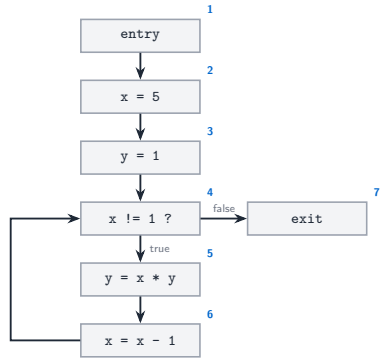

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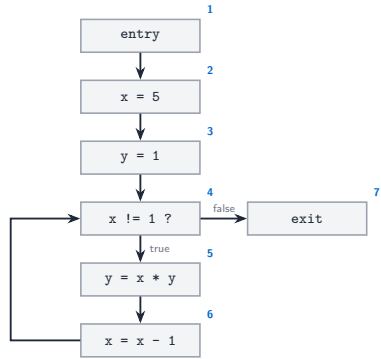

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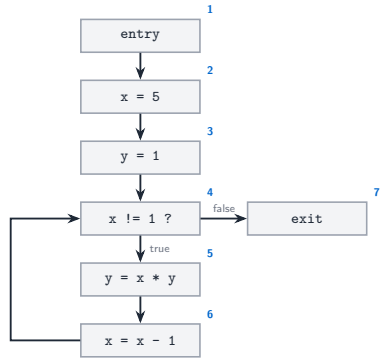

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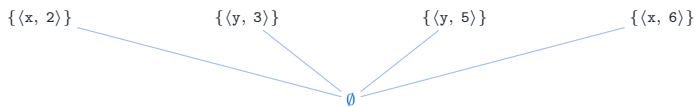

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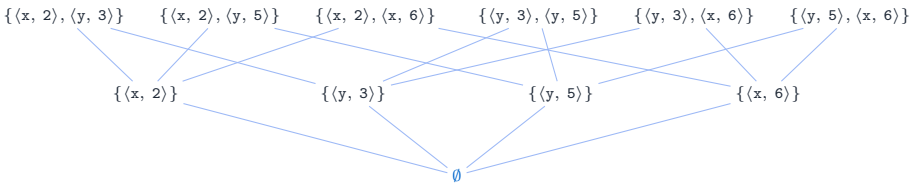

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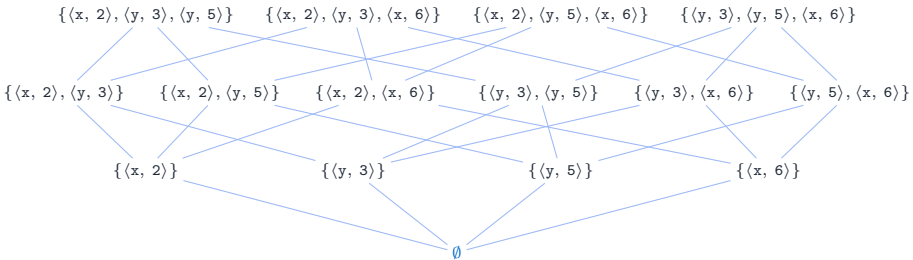

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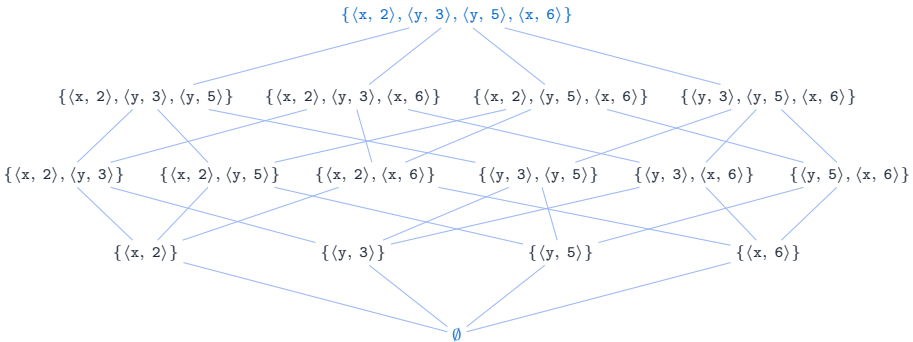

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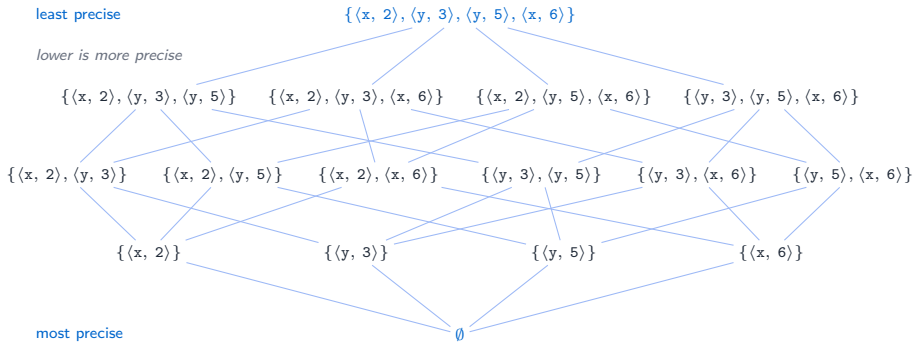
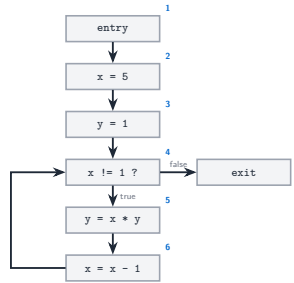


Example

Reaching definitions

Example: Initialisation

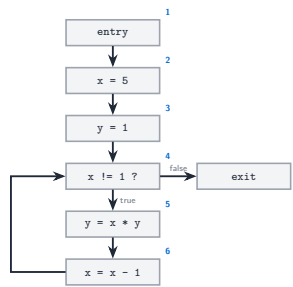
n	**IN**[n]	**OUT**[n]
1	—	
2	$\emptyset$	$\emptyset$
3	$\emptyset$	$\emptyset$
4	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$
7	$\emptyset$	—



Example: Initialisation

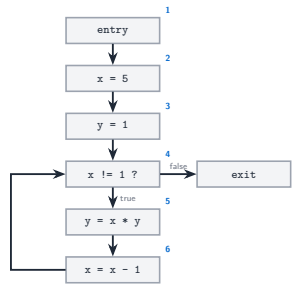
n	**IN**[n]	**OUT**[n]
1	—	$\{\langle \texttt{x}, ? \rangle, \langle \texttt{y}, ? \rangle\}$
2	$\emptyset$	$\emptyset$
3	$\emptyset$	$\emptyset$
4	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$
7	$\emptyset$	—

Only $\mathrm{OUT}[1]$ differs: both variables start uninitialised.



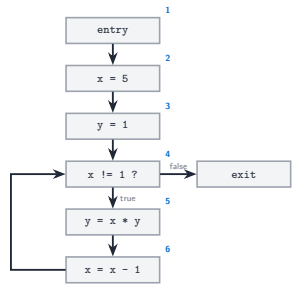
Pass 1

n	IN[n]	OUT[n]
1	__	$\{\langle x, ?\rangle, \langle y, ?\rangle\}$
2		
3		
4		
5		
6		
7		__



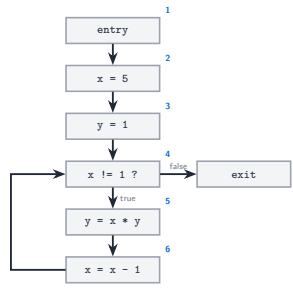
Pass 1

n	IN[n]	OUT[n]
1	__	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3		
4		
5		
6		
7		__



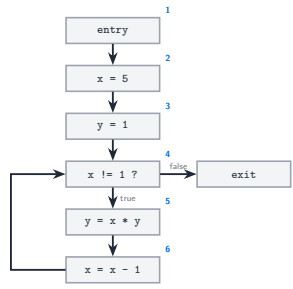
Pass 1

n	IN[n]	OUT[n]
1	__	$\{\langle x, ?\rangle, \langle y, ?\rangle\}$
2	$\{\langle x, ?\rangle, \langle y, ?\rangle\}$	$\{\langle x, 2\rangle, \langle y, ?\rangle\}$
3	$\{\langle x, 2\rangle, \langle y, ?\rangle\}$	$\{\langle x, 2\rangle, \langle y, 3\rangle\}$
4		
5		
6		
7		__



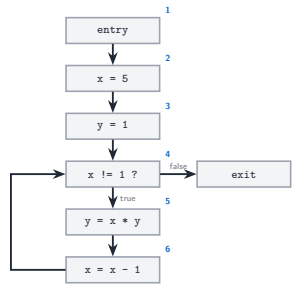
Pass 1

n	IN[n]	OUT[n]
1	__	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
5		
6		
7		__



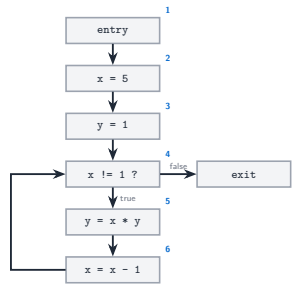
Pass 1

n	IN[n]	OUT[n]
1	__	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
5	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 5⟩}
6		
7		__



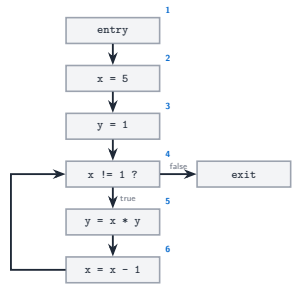
Pass 1

n	IN[n]	OUT[n]
1	__	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
5	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7		__



Pass 1

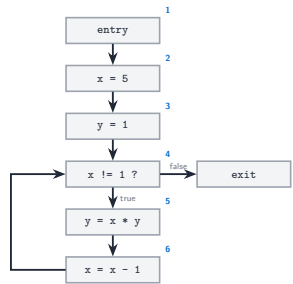
n	IN[n]	OUT[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
5	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7	{⟨x, 2⟩, ⟨y, 3⟩}	—



Pass 1

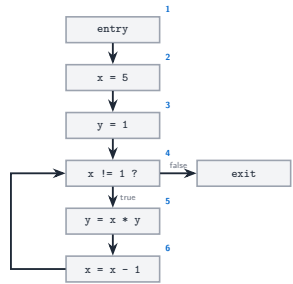
n	IN[n]	OUT[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
5	{⟨x, 2⟩, ⟨y, 3⟩}	{⟨x, 2⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7	{⟨x, 2⟩, ⟨y, 3⟩}	—

Node 6 has not been visited when node 4 is computed, so the loop has not yet contributed anything.



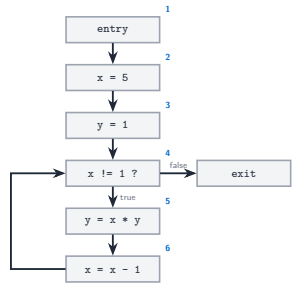
Pass 2

n	**IN**[n]	**OUT**[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4		
5		
6		
7		—



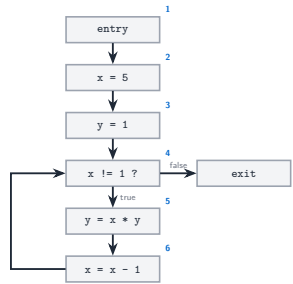
Pass 2

n	**IN**[n]	**OUT**[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	
5		
6		
7		—



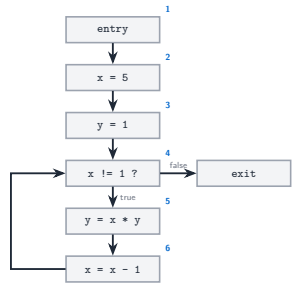
Pass 2

n	IN[n]	OUT[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5		
6		
7		—



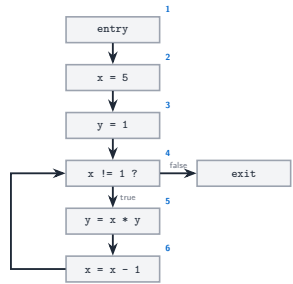
Pass 2

n	**IN**[n]	**OUT**[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}
6		
7		—



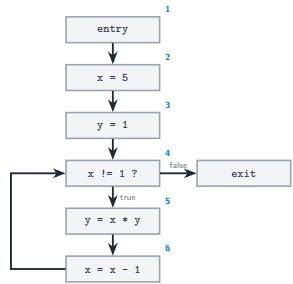
Pass 2

n	**IN**[n]	**OUT**[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7		—



Pass 2

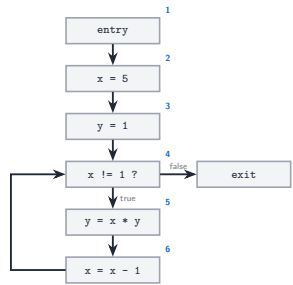
n	**IN**[n]	**OUT**[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	—



Pass 2

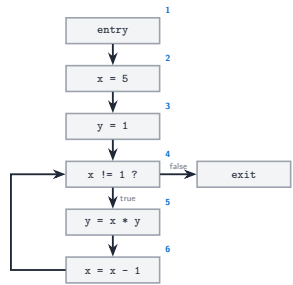
n	IN[n]	OUT[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	—

Red entries changed. OUT[6] did *not*: ⟨x, 2⟩ is killed there either way.



Pass 3: Nothing Changes

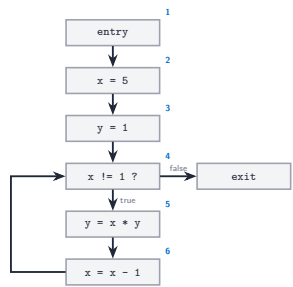
n	**IN**[n]	**OUT**[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	—



Pass 3: Nothing Changes

n	IN[n]	OUT[n]
1	—	{⟨x, ?⟩, ⟨y, ?⟩}
2	{⟨x, ?⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, ?⟩}
3	{⟨x, 2⟩, ⟨y, ?⟩}	{⟨x, 2⟩, ⟨y, 3⟩}
4	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}
5	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}
6	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 5⟩}	{⟨x, 6⟩, ⟨y, 5⟩}
7	{⟨x, 2⟩, ⟨x, 6⟩, ⟨y, 3⟩, ⟨y, 5⟩}	—

Saturated. This is the fixed point.



Reading the Result

- `x = 5` reaches P1: $\langle \mathtt{x}, 2 \rangle \in \mathrm{IN}[4]$ ✓

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Reading the Result

- `x = 5` reaches P1: $\langle \mathtt{x}, 2 \rangle \in \mathrm{IN}[4]$ ✓
- `x = 5` misses P2: $\langle \mathtt{x}, 2 \rangle \notin \mathrm{OUT}[6]$ ✓
- `y = 1` misses P2: $\langle \mathtt{y}, 3 \rangle \notin \mathrm{OUT}[6]$ ✓

Reading the Result

- `x = 5` reaches P1: $\langle \mathtt{x}, 2 \rangle \in \mathrm{IN}[4]$ ✓
- `x = 5` misses P2: $\langle \mathtt{x}, 2 \rangle \notin \mathrm{OUT}[6]$ ✓
- `y = 1` misses P2: $\langle \mathtt{y}, 3 \rangle \notin \mathrm{OUT}[6]$ ✓
- `y = x * y` reaches both: $\langle \mathtt{y}, 5 \rangle \in \mathrm{IN}[4]$ and $\mathrm{OUT}[6]$ ✓

Reading the Result

- `x = 5` reaches P1: $\langle \mathtt{x}, 2 \rangle \in \mathrm{IN}[4]$ ✓
- `x = 5` misses P2: $\langle \mathtt{x}, 2 \rangle \notin \mathrm{OUT}[6]$ ✓
- `y = 1` misses P2: $\langle \mathtt{y}, 3 \rangle \notin \mathrm{OUT}[6]$ ✓
- `y = x * y` reaches both: $\langle \mathtt{y}, 5 \rangle \in \mathrm{IN}[4]$ and $\mathrm{OUT}[6]$ ✓

Application: Uninitialised Variables

- Variable v read at node n *may* be uninitialised $\iff \langle \mathrm{v}, ? \rangle \in \mathrm{IN}[n]$.

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- $\langle \mathrm{x}, ? \rangle \notin \mathrm{IN}[4] \Rightarrow$ **x is always initialised there.** *Proved.*

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- $\langle \mathrm{x}, ? \rangle \notin \mathrm{IN}[4] \Rightarrow$ **x is always initialised there.** *Proved.*

Absence of the fact is a **proof** — soundness makes it meaningful.

Presence of the fact is only a **warning** — the path may be infeasible.