L18: DFA: Very Busy, Available, and Live

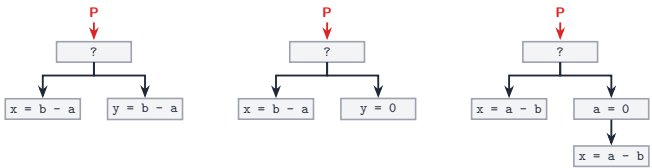
Software Analysis

IIT Guwahati · September 9, 2026

Very Busy Expression Analysis

What Makes an Expression Very Busy?

Two things must hold on *every* path leaving the point: the expression is **used**, and it is used **before** any of its variables changes.



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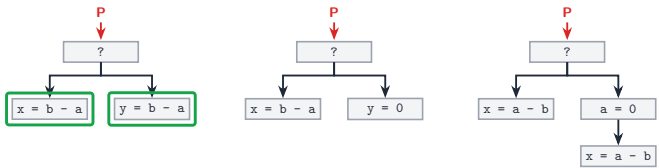

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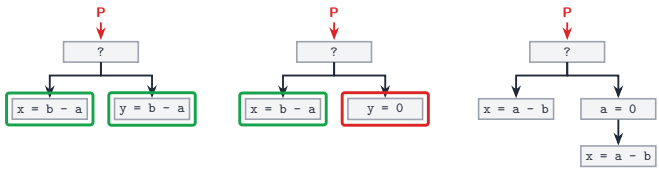

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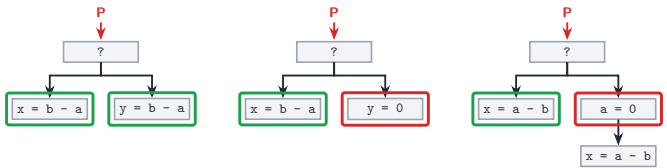

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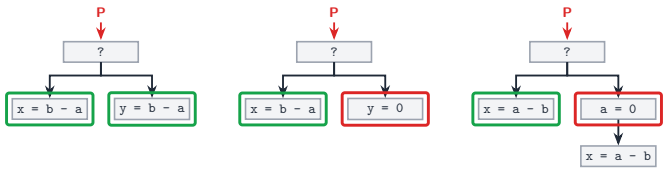
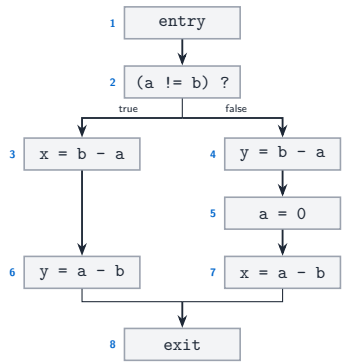


Only the first is very busy at **P**. In the third, `a - b` *is* recomputed later: it is a *different* value.

Goal: Very Busy Expressions

An expression is **very busy** at a program point if, *no matter which path is taken*, it is always used before any of the variables occurring in it is redefined.



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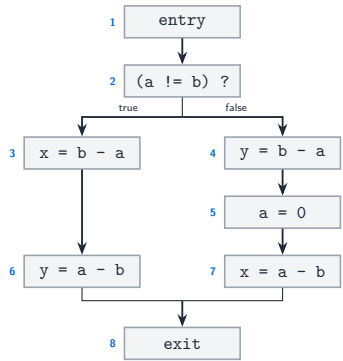

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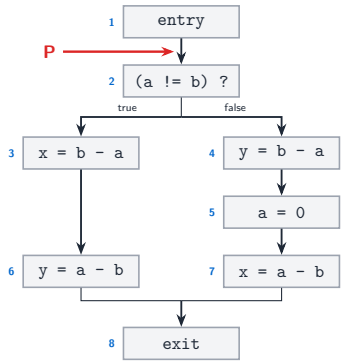
Consider point **P**, the entry of node 2.



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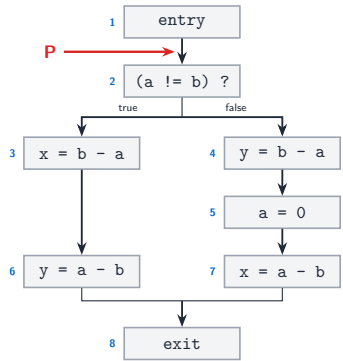

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`b − a` is very busy at P.

Used at node 3 on the true path,
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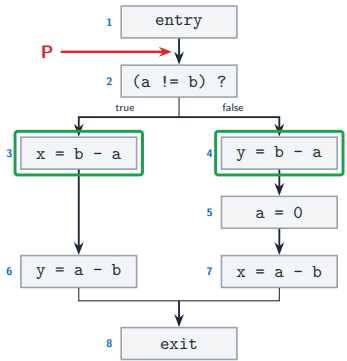
Neither `a` nor `b` changes first.



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Used at node 6 on the true path ✓
but on the false path node 5 sets a = 0
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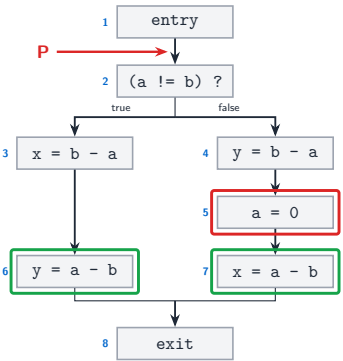

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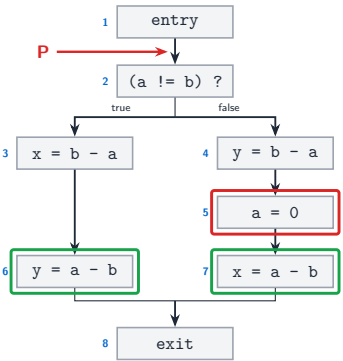

One bad path is enough to disqualify an expression — that is what **must** means.

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$$\mathrm{OUT}[n] = \bigcap_{n' \in succ(n)} \mathrm{IN}[n']$$

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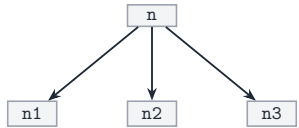

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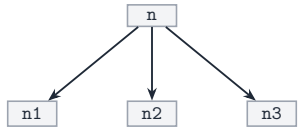

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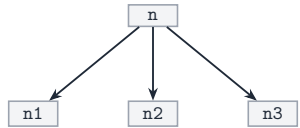

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Two changes from reaching definitions: **successors** not predecessors, and $\cap$ not $\cup$.

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- Same shape as before, with **IN and OUT swapped**.

Very Busy: the GEN and KILL Rules

Statement at n	**GEN**$[n]$	**KILL**$[n]$
a condition, e.g. $(\mathtt{a} != \mathtt{b})$		
an assignment $\mathtt{x} = \mathtt{a}$		

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An assignment **uses** its right-hand side and **invalidates** everything mentioning its left-hand side.

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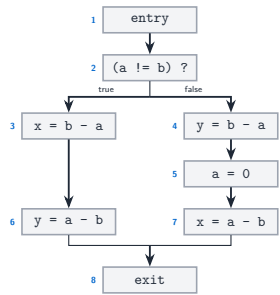

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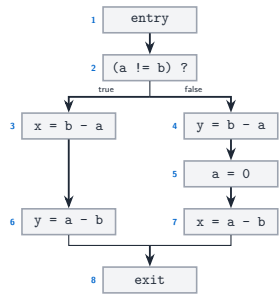

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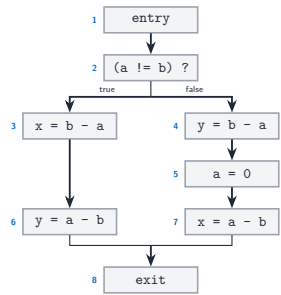

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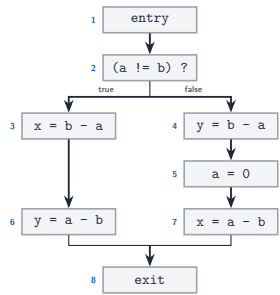

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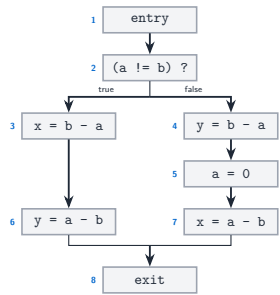

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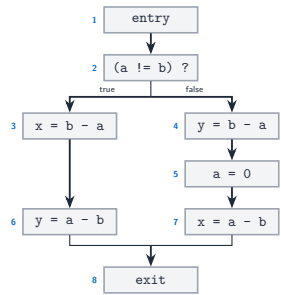

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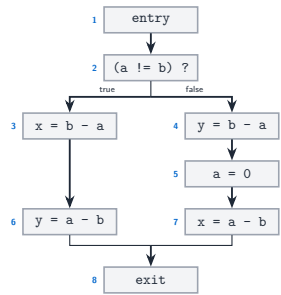

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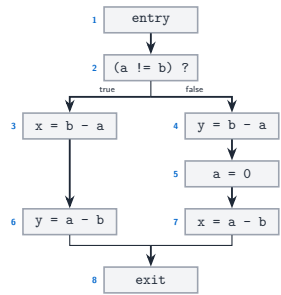
Only node 5 kills anything — and it kills *both*, since a occurs in both expressions.



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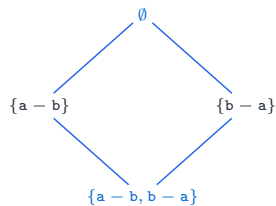

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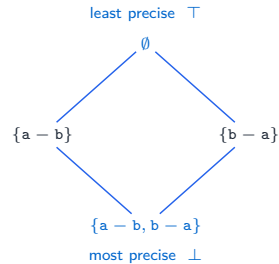

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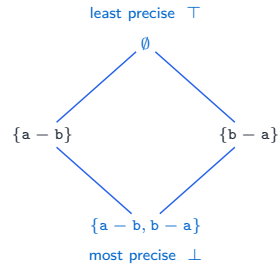
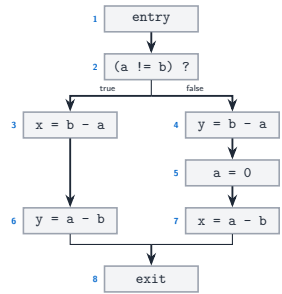


Very Busy: Initialisation

Every set starts as $\{a - b, b - a\}$, all the expressions.

n	**IN**$[n]$	**OUT**$[n]$
1	—	$\{a - b, b - a\}$
2	$\{a - b, b - a\}$	$\{a - b, b - a\}$
3	$\{a - b, b - a\}$	$\{a - b, b - a\}$
4	$\{a - b, b - a\}$	$\{a - b, b - a\}$
5	$\{a - b, b - a\}$	$\{a - b, b - a\}$
6	$\{a - b, b - a\}$	$\{a - b, b - a\}$
7	$\{a - b, b - a\}$	$\{a - b, b - a\}$
8		—



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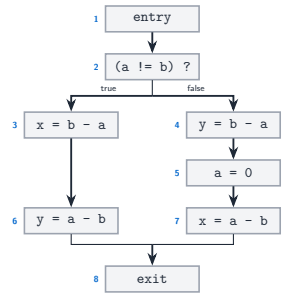
7	$\{a - b, b - a\}$	$\{a - b, b - a\}$
8	$\emptyset$	—

Only IN[8] differs: nothing is very busy at the end.



Very Busy: Pass 1

Visiting nodes $7, 6, 5, 4, 3, 2, 1$.

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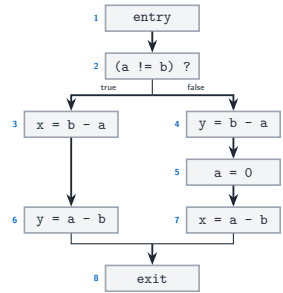

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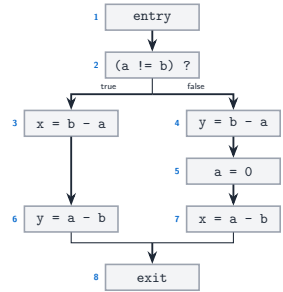

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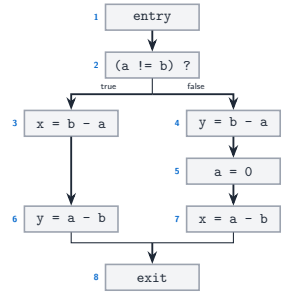

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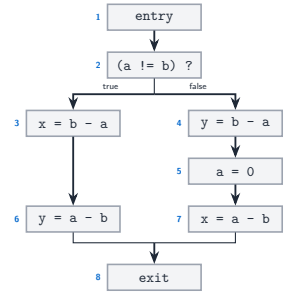

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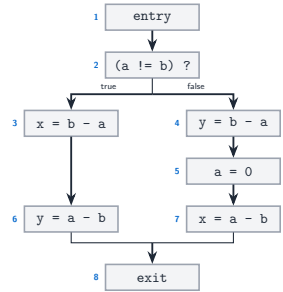

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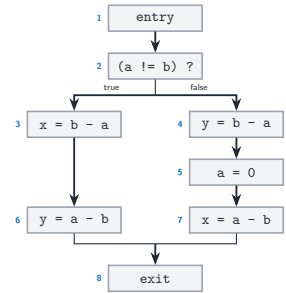

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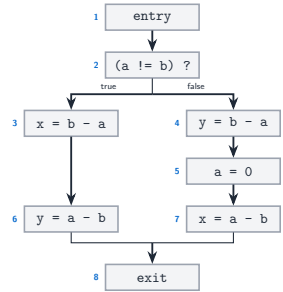

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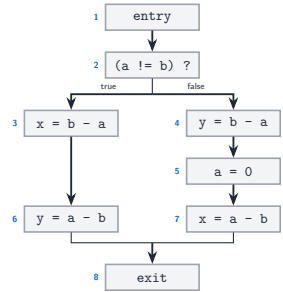

Node 5 wipes both expressions, so $a - b$ never escapes the false branch.

Very Busy: Pass 2 — Saturated

n	**IN**$[n]$	**OUT**$[n]$
1	—	$\{b - a\}$
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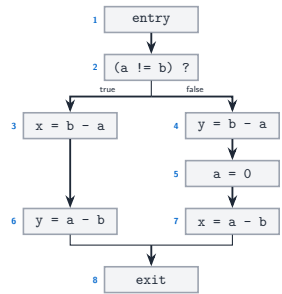

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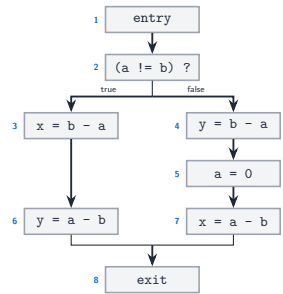
Saturated. This is the fixed point.



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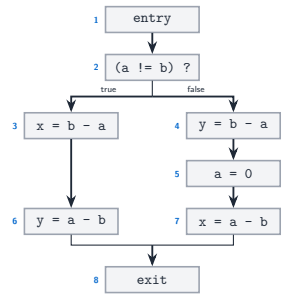

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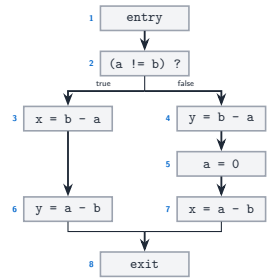

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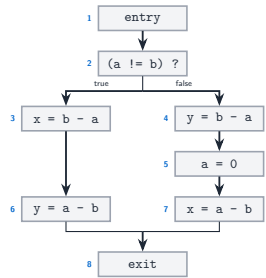

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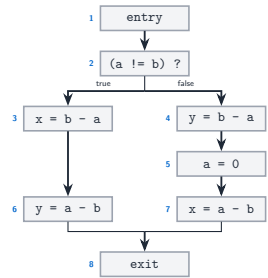

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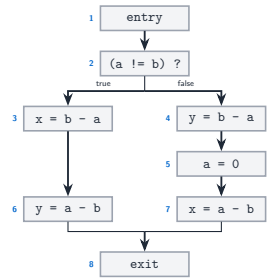

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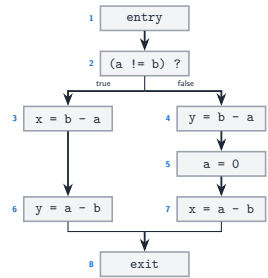
Table: they are $\mathrm{IN}[3]$, $\mathrm{IN}[5]$ and $\mathrm{IN}[7]$.



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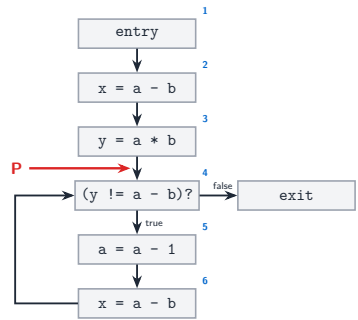

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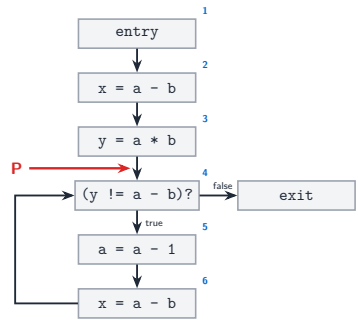

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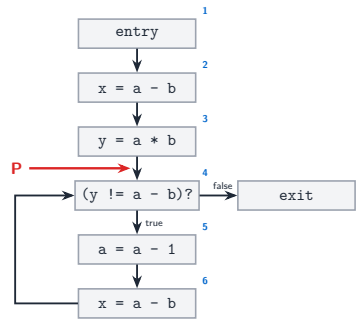
Consider **P**, the entry of node 4.



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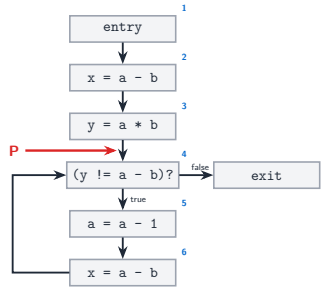

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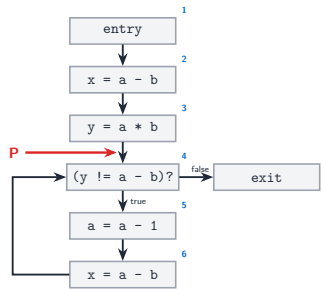

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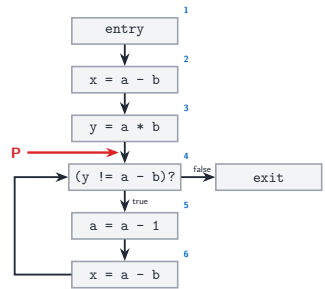

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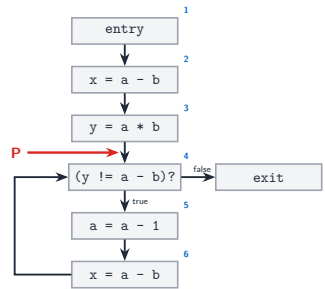

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An expression is available at n only if it is available on *every* way of getting there.

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It still kills every expression using a — here, all three.

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n	Statement	GEN	KILL
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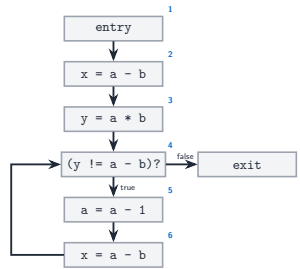

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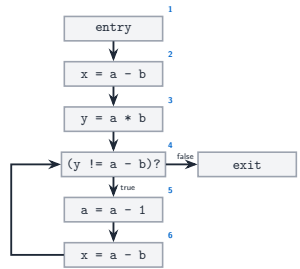

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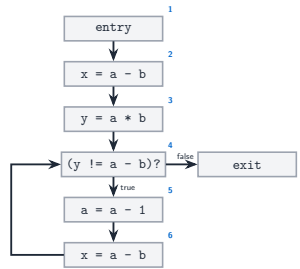

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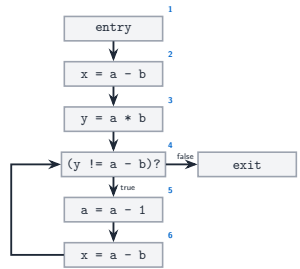
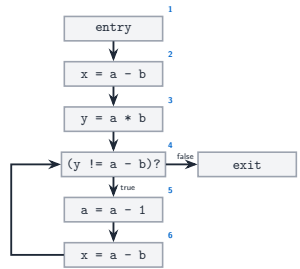
5	$\mathtt{a} = \mathtt{a} - 1$		
6	$\mathtt{x} = \mathtt{a} - \mathtt{b}$		



Available: GEN and KILL for the Program

n	Statement	GEN	KILL
2	$\mathtt{x} = \mathtt{a} - \mathtt{b}$	$\{\mathtt{a} - \mathtt{b}\}$	$\emptyset$
3	$\mathtt{y} = \mathtt{a} * \mathtt{b}$	$\{\mathtt{a} * \mathtt{b}\}$	$\emptyset$
4	$(\mathtt{y} != \mathtt{a} - \mathtt{b})?$	$\emptyset$	$\emptyset$
5	$\mathtt{a} = \mathtt{a} - 1$	$\emptyset$	$\{\mathtt{a} - \mathtt{b}, \mathtt{a} * \mathtt{b}, \mathtt{a} - 1\}$
6	$\mathtt{x} = \mathtt{a} - \mathtt{b}$		



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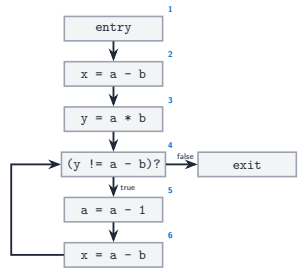
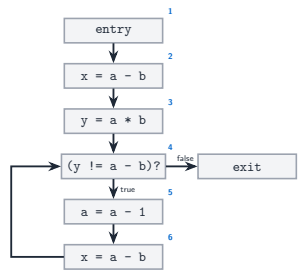
6	$\mathtt{x} = \mathtt{a} - \mathtt{b}$	$\{\mathtt{a} - \mathtt{b}\}$	$\emptyset$



Available: GEN and KILL for the Program

n	Statement	GEN	KILL
2	$\mathtt{x} = \mathtt{a} - \mathtt{b}$	$\{\mathtt{a} - \mathtt{b}\}$	$\emptyset$
3	$\mathtt{y} = \mathtt{a} * \mathtt{b}$	$\{\mathtt{a} * \mathtt{b}\}$	$\emptyset$
4	$(\mathtt{y} != \mathtt{a} - \mathtt{b})?$	$\emptyset$	$\emptyset$
5	$\mathtt{a} = \mathtt{a} - 1$	$\emptyset$	$\{\mathtt{a} - \mathtt{b}, \mathtt{a} * \mathtt{b}, \mathtt{a} - 1\}$
6	$\mathtt{x} = \mathtt{a} - \mathtt{b}$	$\{\mathtt{a} - \mathtt{b}\}$	$\emptyset$

$\mathtt{a} - 1$ is killed at node 5 and generated *nowhere* — so it is never available at any point.



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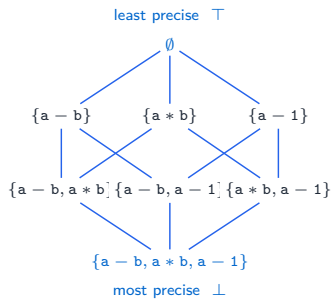

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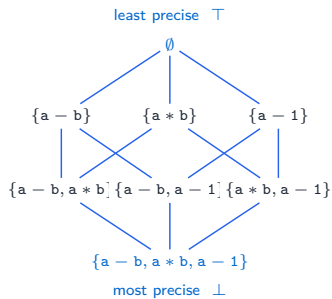

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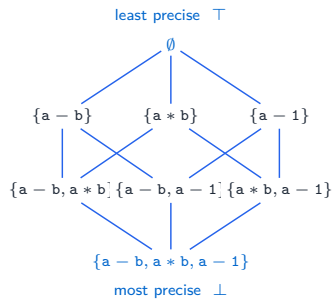
Sets start *full* and **shrink**. Boundary: $\mathrm{OUT}[\mathit{entry}] = \emptyset$.



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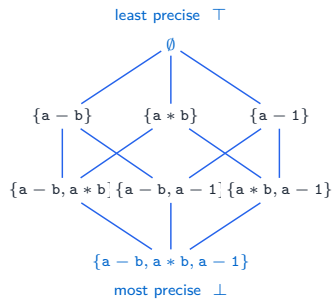

Available: Pass 1 — Forward

Init: all $\{a - b, a * b, a - 1\}$, except $\mathrm{OUT}[1] = \emptyset$. Visiting $2 \ldots 7$.

n	**IN**$[n]$	**OUT**$[n]$
1	—	$\emptyset$
2	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
3	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
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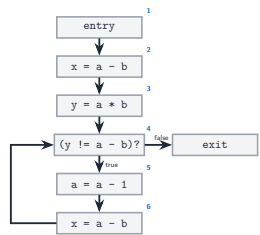
6	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
7	$\{a - b, a * b, a - 1\}$	—



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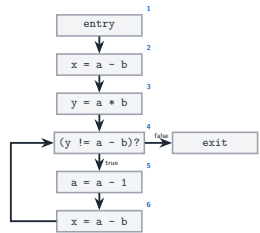

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n	**IN**$[n]$	**OUT**$[n]$
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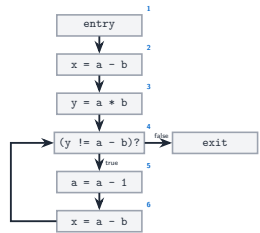
4	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
5	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
6	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
7	$\{a - b, a * b, a - 1\}$	—



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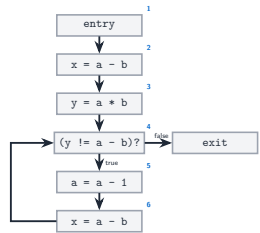
4	$\{a - b, a * b\}$	$\{a - b, a * b\}$
5	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
6	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
7	$\{a - b, a * b, a - 1\}$	—



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6	$\{a - b, a * b, a - 1\}$	$\{a - b, a * b, a - 1\}$
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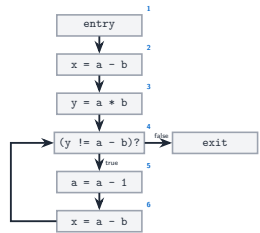

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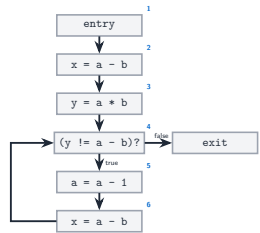

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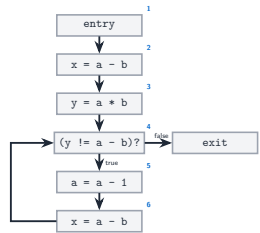

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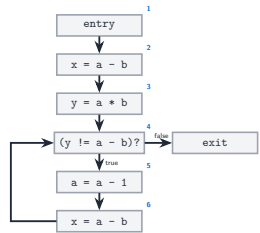
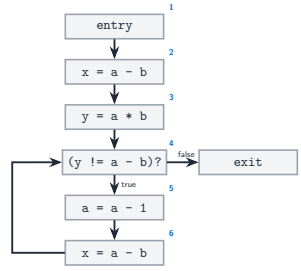


At node 4 we intersected with $\mathrm{OUT}[6]$, which was still the full set — so this answer is not final.

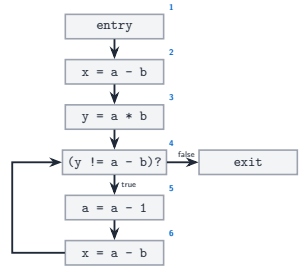
Available: Pass 2

n	IN[n]	OUT[n]
1	—	$\emptyset$
2	$\emptyset$	$\{a - b\}$
3	$\{a - b\}$	$\{a - b, a * b\}$
4	$\{a - b, a * b\}$	$\{a - b, a * b\}$
5	$\{a - b, a * b\}$	$\emptyset$
6	$\emptyset$	$\{a - b\}$
7	$\{a - b, a * b\}$	—



Available: Pass 2

n	IN[n]	OUT[n]
1	—	$\emptyset$
2	$\emptyset$	$\{a - b\}$
3	$\{a - b\}$	$\{a - b, a * b\}$
4	$\{a - b\}$	$\{a - b\}$
5	$\{a - b, a * b\}$	$\emptyset$
6	$\emptyset$	$\{a - b\}$
7	$\{a - b, a * b\}$	—



$$\text{IN}[4] = \text{OUT}[3] \cap \text{OUT}[6] =$$

$$\{a - b, a * b\} \cap \{a - b\}$$

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n	IN[n]	OUT[n]
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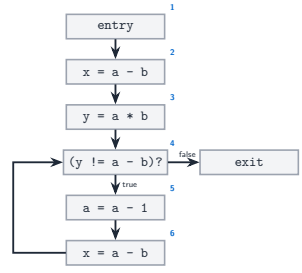

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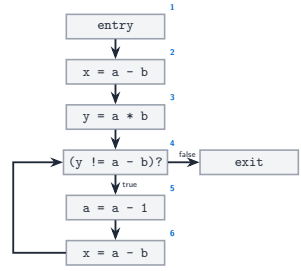

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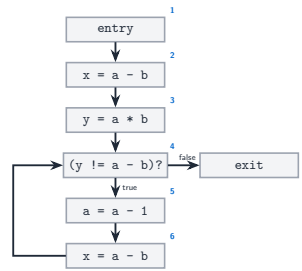
Pass 3 changes nothing — this is the fixed point.



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$\mathtt{a} - \mathtt{b}$ is available at nodes 4, 5 and 7 — everywhere the loop condition can be reached.

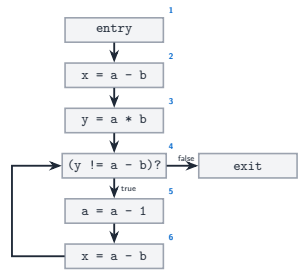
Quiz: Available Expressions

Which expressions are available at the stated points?

1. a − b at the entry of node 3?

2. a ∗ b at the entry of node 5?

3. Anything at all at the entry of node 6?



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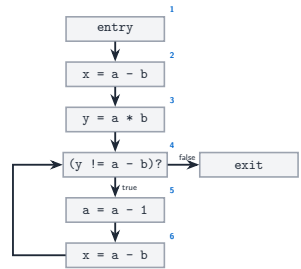

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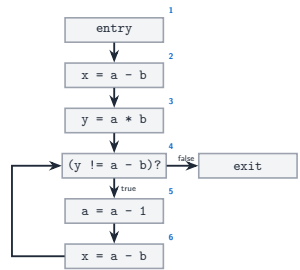

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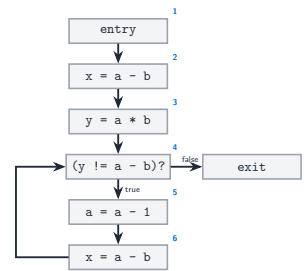

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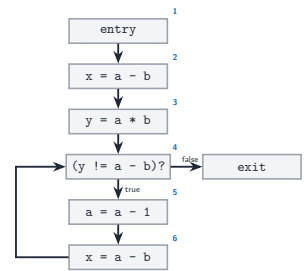
   NO
3. Anything at all at the entry of node 6? **NO**

a − 1 is available *nowhere* — no node ever generates it.



Application: Avoiding Recomputation

If an expression is available at a point, its value has already been computed on every path here.

So do not compute it again — **reuse** it.

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- **Common subexpression elimination.** The program gets *faster*.

Application: Avoiding Recomputation

If an expression is available at a point, its value has already been computed on every path here.

So do not compute it again — **reuse** it.

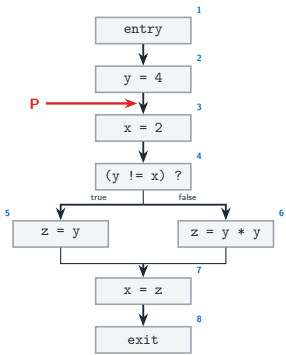
- $\mathtt{a} - \mathtt{b}$ is available at node 4, which tests $\mathtt{y} != \mathtt{a} - \mathtt{b}$.
- That test can reuse the value computed at node 2 or node 6 instead of recomputing it.
- **Common subexpression elimination.** The program gets *faster*.

Contrast very busy expressions, which saved code *size*. This one saves *time*.

Live Variable Analysis

What Makes a Variable Live?

A variable is **live** at a program point if there is a path from that point to a *use* of the variable which does not redefine it first.



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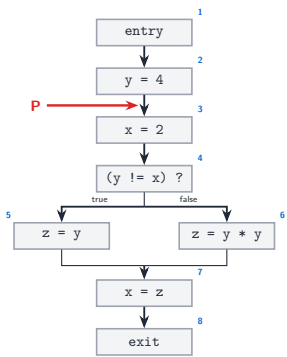

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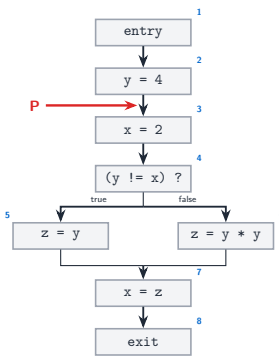

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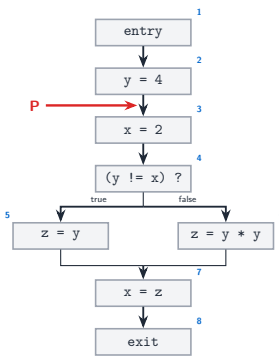
"Live" means *still needed*. A dead variable's value can be thrown away.



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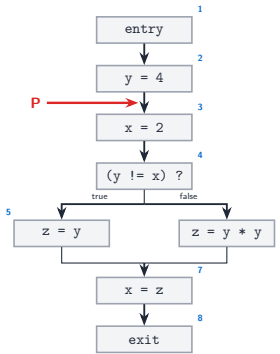

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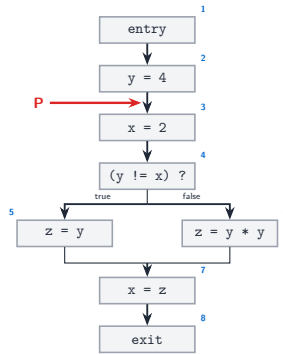

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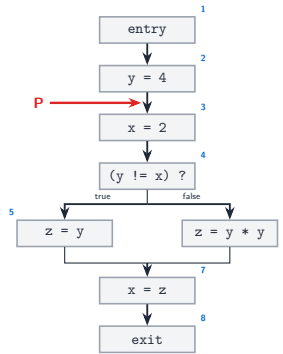

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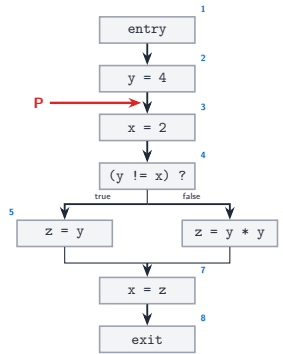

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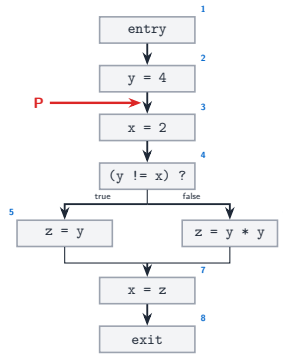

For z, *every* path redefines it first — so even a *may* analysis rules it out.

The Two Operations, Reversed

1 — Merge at the exit (backward, so successors)

$$\mathrm{OUT}[n] = \bigcup_{n' \in succ(n)} \mathrm{IN}[n']$$

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Live: GEN and KILL — Reads and Writes

n	Statement	GEN (used)	KILL (defined)
2	$y = 4$		
3	$x = 2$		
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5	$z = y$		
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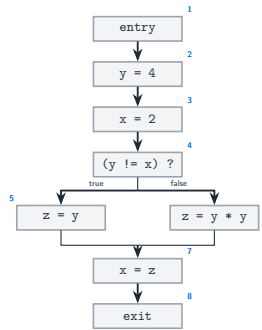

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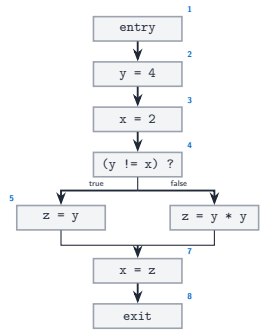

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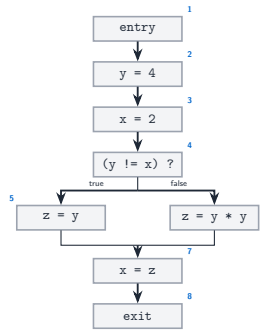

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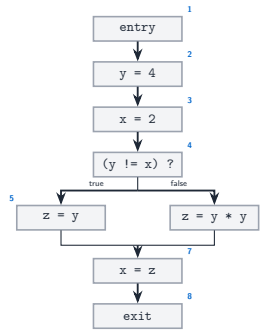

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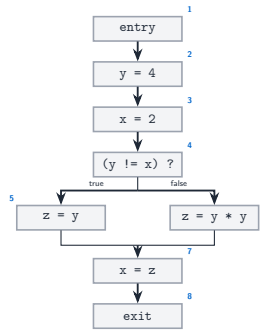

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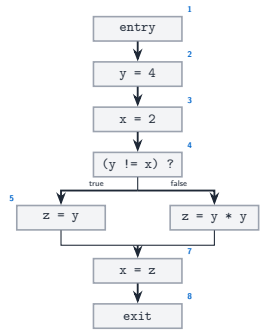

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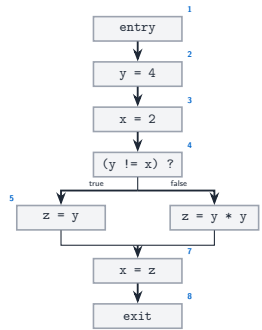

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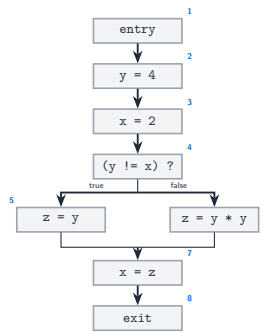
A **condition** generates but never kills — it reads and writes nothing.



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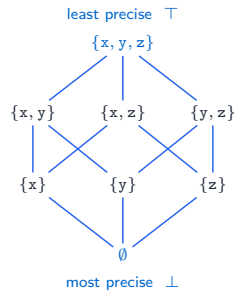

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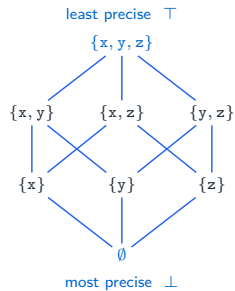

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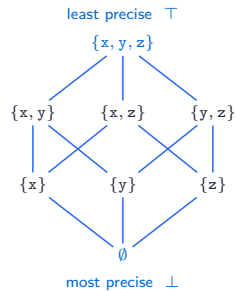
Sets start *empty* and **grow**. Boundary: $\mathrm{IN}[\mathit{exit}] = \emptyset$.



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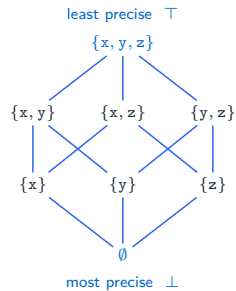
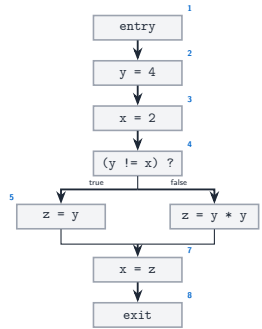
A *smaller* set of live variables is a stronger claim, so smaller is more precise.



Live: Pass 1 — Working Backwards

Init: all $\emptyset$. Visiting $7, 6, 5, 4, 3, 2, 1$.

n	**IN**[n]	**OUT**[n]
1	—	$\emptyset$
2	$\emptyset$	$\emptyset$
3	$\emptyset$	$\emptyset$
4	$\emptyset$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$\emptyset$
7	{z}	$\emptyset$
8	$\emptyset$	—



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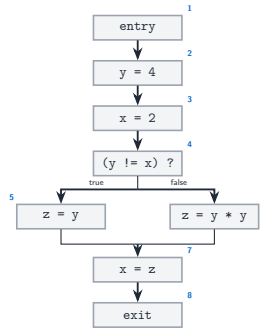

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4	$\emptyset$	$\emptyset$
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7	$\{z\}$	$\emptyset$
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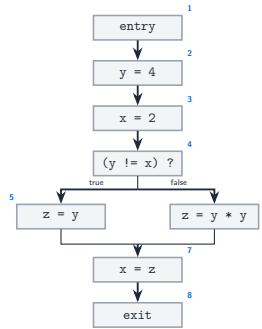

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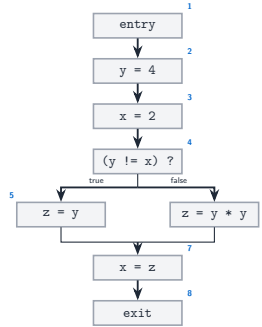

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n	**IN**$[n]$	**OUT**$[n]$
1	—	$\emptyset$
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3	$\{\texttt{y}\}$	$\{\texttt{x}, \texttt{y}\}$
4	$\{\texttt{x}, \texttt{y}\}$	$\{\texttt{y}\}$
5	$\{\texttt{y}\}$	$\{\texttt{z}\}$
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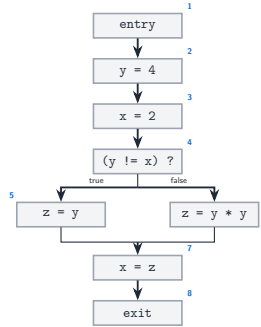

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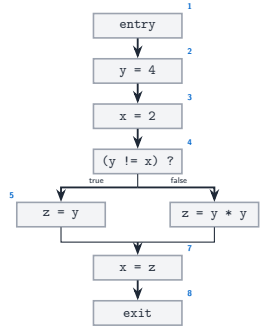

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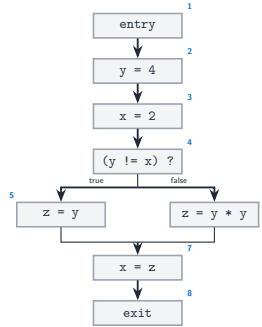
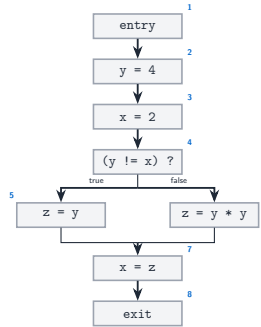


Node 4 is where the two branches rejoin — and both contribute y.

Live: Pass 2 — Saturated

n	**IN**[n]	**OUT**[n]
1	—	$\emptyset$
2	$\emptyset$	$\{y\}$
3	$\{y\}$	$\{x, y\}$
4	$\{x, y\}$	$\{y\}$
5	$\{y\}$	$\{z\}$
6	$\{y\}$	$\{z\}$
7	$\{z\}$	$\emptyset$
8	$\emptyset$	—



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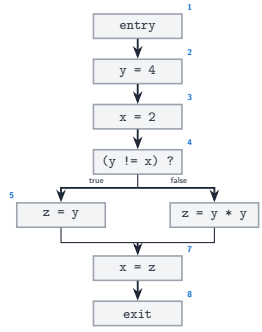
n	**IN**[n]	**OUT**[n]
1	—	$\emptyset$
2	$\emptyset$	$\{\mathtt{y}\}$
3	$\{\mathtt{y}\}$	$\{\mathtt{x}, \mathtt{y}\}$
4	$\{\mathtt{x}, \mathtt{y}\}$	$\{\mathtt{y}\}$
5	$\{\mathtt{y}\}$	$\{\mathtt{z}\}$
6	$\{\mathtt{y}\}$	$\{\mathtt{z}\}$
7	$\{\mathtt{z}\}$	$\emptyset$
8	$\emptyset$	—

Saturated. This is the fixed point.



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n	**IN**[n]	**OUT**[n]
1	—	$\emptyset$
2	$\emptyset$	$\{y\}$
3	$\{y\}$	$\{x, y\}$
4	$\{x, y\}$	$\{y\}$
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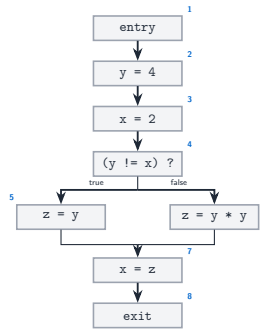
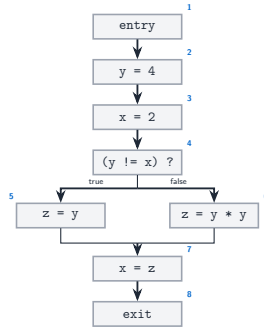


At no point are more than two variables live at once.

Quiz: Live Variables

Which variables are live at the stated points?

1. x at the entry of node 4?
2. z at the entry of node 5?
3. y at the exit of node 7?



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Which variables are live at the stated points?

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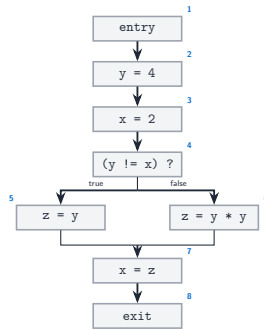

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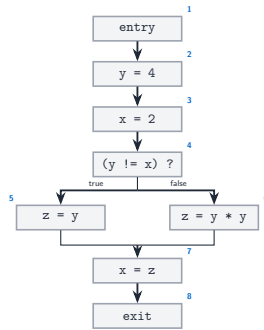

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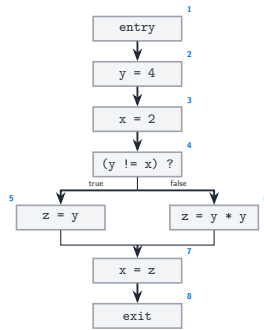

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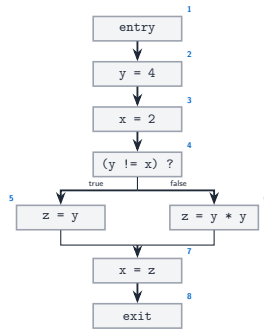

Nothing is live at the exit — the program is over, so no value is needed.

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Fewer registers means less spilling to memory — and register allocation is the compiler stage that most affects the speed of the generated code.

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backward		

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Summary

Analysis	Dir.	Kind	Merge	$\perp$	Application
Reaching definitions	fwd	may	$\cup$	$\emptyset$	uninitialised variables
Very busy expressions	bwd	must	$\cap$	all	reduce code size
Available expressions	fwd	must	$\cap$	all	avoid recomputation
Live variables	**bwd**	**may**	$\cup$	$\emptyset$	**register allocation**